Universality class of discrete solid-on-solid limited mobility nonequilibrium growth models for kinetic surface roughening

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We investigate, using the noise reduction technique, the *asymptotic* universality class of the well-studied nonequilibrium limited mobility atomistic solid-on-solid surface growth models introduced by Wolf and Villain (WV) and Das Sarma and Tamborenea (DT) in the context of kinetic surface roughening in ideal molecular beam epitaxy. We find essentially all the earlier conclusions regarding the universality class of DT and WV models to be severely hampered by slow crossover and extremely long-lived transient effects. We identify the correct asymptotic universality class(es) that differs from earlier conclusions in several instances.

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Kinetic surface roughening of nonequilibrium growth models, particularly under solid-on-solid (SOS) growth conditions, remains a subject of considerable interest and activity in spite of a great deal of theoretical and experimental research in the topic during the last decade [1]. Solid-on-solid or SOS growth is an extensively used lattice growth model where each atom can only sit on top of another atom so that overhangs and vacancies are not allowed. While the *generic* nonequilibrium surface growth universality class (in the situation allowing overhangs and bulk defects, i.e., under generic non-SOS conditions) is theoretically accepted to be the Kardar-Parisi-Zhang (KPZ) universality (although the asymptotic KPZ universality in specific models may be masked by extremely slow crossover effects), there is no such consensus for SOS growth models where seemingly innocuous small changes in local growth rules appear to lead to different dynamic universality classes. In fact, some of these nonequilibrium SOS growth models, introduced in the context of mimicking ideal molecular beam epitaxy (MBE), do not even seem to obey self-affine dynamic scaling behavior, instead exhibiting nonuniversal anomalous and multi-affine scaling. It may, therefore, be questioned whether the universality class concept is useful in SOS growth models or even does it apply to nonequilibrium SOS growth models. We note that in MBE growth one tries to avoid as much as possible the formation of overhangs and vacancies or void in the growing film in order to obtain high-quality thin films, and therefore SOS conditions of no overhangs and vacancies apply quite accurately to MBE growth.

In this paper we address this confusing situation plaguing our understanding of the universality class of discrete SOS nonequilibrium growth models by concentrating on a specific class of surface diffusion driven stochastic growth models that mimic in a drastically simplified manner low-temperature molecular beam epitaxial growth. This class of models has been referred to as "limited mobility" nonequilibrium growth models because the deposited atoms are only allowed to diffuse (obeying certain specific local diffusion rules) at incidence, and all the other atoms in the growing film, except for the most recently incident atom, do not diffuse. In spite of their highly simplified nature, limited mobility nonequilibrium surface growth models have attracted a great deal of attention [1] primarily because of the following three reasons.

(1) These are the first growth models that were demonstrated to lie outside the generic universality class,

(2) In spite of their highly simplified nature, the various growth exponents and growth morphologies in these models seem to agree well with those in full diffusion Arrhenius hopping MBE growth models (where all atoms at the growth front are allowed to hop continuously according to temperature-dependent hopping rates) in the low-temperature kinetically rough regime.

(3) Obtaining a coarse-grained *continuum* description of the *discrete* low mobility growth models (i.e., writing down a continuum dynamical growth equation corresponding to the cellular automata discrete rules defining the low mobility growth models) has turned out to be extremely difficult in spite of the deceptive simplicity of the models. The innovation and the technique we introduce in the study of these extremely well-studied limited mobility growth models is the use of the *noise reduction* technique [2] in the direct numerical simulation of the atomistic growth rules. Our conclusion, based on extensive numerical simulations in both $1+1$ and $2+1$ dimensions (plus some limited simulations in higher dimensions) is that the *asymptotic* universality class of the various limited mobility growth models is a surprisingly subtle issue with many of the earlier findings (including some from our group) being *incorrect* due to pathologically slow crossover and extremely long transient effects that typically distract from ascertaining the true asymptotic universality class of these models.

One striking feature of our results is the apparent dependence of the asymptotic universality class on the system dimensionality d, not merely in the sense of well-known hyperscaling relations, but in the fact that the applicable hyperscaling relation itself (connecting the dynamical exponent z with the roughness exponent α for example), for a specific limited mobility growth model, may depend on d. We believe that such dimension-dependent *nonuniversal* universality, where the hyperscaling relation for a given model changes with d (and which, to the best of our knowledge, has

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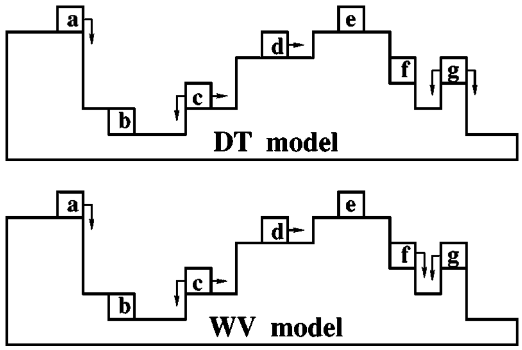

FIG. 1. Schematic configurations defining the growth rules for the DT and WV models in $d=1+1$. For the ADT model, adatom "g" in the top (DT) plot will only hop to the left where it will have three nearest neighbors. For the SWV model, adatom "g" in the bottom (WV) plot will choose between the left and right neighbors with equal probability.

no known analog in equilibrium critical phenomena where a given Hamiltonian or free energy functional, e.g., the ϕ^4 model, is characterized by a unique hyperscaling relation that *does not* change with the specific value of d), is a characteristic feature of nonequilibrium processes, and may transcend the specific models and phenomena being studied here. One implication of this peculiar dimension-dependent universality is that the universality class concept, which is one of the most important conceptual foundations of modern critical phenomena, may be of rather limited validity and usefulness in nonequilibrium phenomena since the model by itself does not specify the universality class—it depends on the model (i.e., the specific set of discrete dynamic growth rules in our growth model) *and also on the dimensionality*.

We study the limited mobility nonequilibrium growth models introduced by Wolf and Villain (WV) [3] and by Das Sarma and Tamborenea (DT) [4], both of which have already been studied extensively in the literature and discussed rather elaborately in recent reviews [5]. We also study two simple variants of these models, which we call *asymmetric* DT (ADT) and *symmetric* WV (SWV) models. The growth rules for these models are described in Fig. 1 for $d=1+1$—the rules for higher dimensions involve straightforward generalization of the rules depicted in Fig. 1. In all four models, atoms are deposited randomly on a $(d-1)$-dimensional square lattice substrate (which is flat initially) obeying the SOS constraint—a deposited atom is then allowed to diffuse or hop instantaneously at incidence (all limited mobility models are also by definition, *instantaneous relaxation* models) to its final incorporation site (after which the *incorporated* atom never moves again) according to the bonding configurations of the deposition site. In DT(WV) model the deposited atom tries to increase (maximize) its nearest-neighbor bonding configuration by moving to the incorporation site. If the deposition and all possible incorporation sites have the same nearest-neighbor bonding configurations, the incident atom does not move and stays at the site of deposition. The diffusion length l, the distance over which the deposited atom is allowed to search to find its incorporation site, is a parameter of the model, and the simulation results shown in this paper use $l=1$ (with the length unit being the lattice constant throughout this paper), i.e., nearest-neighbor diffusion only. We have, however, verified that l is an irrelevant variable, and all our conclusions in this paper are independent of the precise value of l as long as $l \ll L$, where L is the linear size of the substrate, although finite size and crossover effects may be strongly (and nontrivially) dependent on the value of l/L. The DT model has the additional diffusion constraint that only deposited atoms with no lateral nearest-neighbor bonds (i.e., the random incidence site has no occupied nearest neighbor in the same "layer") are allowed to move, whereas in the WV model *all* deposited atoms may move provided the local coordination number is maximized. The ADT and the SWV models are intermediate in their dynamics with respect to DT and WV models in the sense that ADT maximizes the local coordination while still allowing only deposited atoms with no nearest-neighbor lateral bonds to diffuse, whereas the SWV model only increases the local coordination while allowing all deposited atoms to move (provided they can increase their local coordination or nearest-neighbor bonding configuration). In all the models, the incident atom is allowed to move randomly to the incorporation site if several possible incorporation sites satisfy the growth rules.

All our simulations utilize the noise reduction technique introduced by two of us in an earlier paper [2]. The noise reduction technique allows only a fraction of the successful hits (i.e., the incident atoms that satisfy the specific growth rules of the model) to be executed—the noise reduction parameter m defines the number of successful hits needed before incorporation is allowed ($m=1$ is the original DT or WV model). It is well-known that the noise reduction technique, which has been extremely successful in clarifying the universality class of various non-SOS growth models (e.g., Eden model, DLA) with severe crossover problems, is very effective in obtaining the universality class of growth models. In particular, the noise reduction technique ($m>1$) effectively suppresses the severe correction to the scaling induced by the strong stochasticity in limited mobility growth models, and thus makes it easier to determine the asymptotic universality class. We emphasize that the noise reduction technique is absolutely crucial in obtaining the results and conclusions presented in this paper, and its introduction in the simulations is the key feature that enables us to determine the universality class(es) of the limited mobility growth models we study in this paper.

The idea behind the noise reduction technique, which in the context of limited mobility nonequilibrium SOS growth models (being discussed in this paper) has been described in detail in Ref. [2], is that noise induces large hills and valleys on the growing surface that hinder the accurate measurement of the growth exponents, and consequently any technique that suppresses large hills and valleys (i.e., large nearest-neighbor height differences) should lead to better scaling behavior and relatively more accurate estimates of the growth exponents. The noise reduction technique, which has been very successful in suppressing corrections to the scaling in several different nonequilibrium growth processes (e.g., diffusion-limited aggregation, Eden growth, ballistic deposition), achieves good scaling behavior (and consequently better critical exponents) by accepting only a small fraction (to be precise, a fraction of $1/m$, where $m>1$ is the noise reduction factor) of the incident particles and thereby greatly suppressing large hills and valleys in the noise-reduced growth morphology. Earlier detailed analysis by two of us [2] established the noise reduction technique as a highly effective tool in eliminating corrections to scaling and crossover effects in the dynamical growth simulations of discrete SOS limited mobility nonequilibrium growth models for kinetic surface roughening (e.g., DT and WV models) of interest to us in this paper. The basic algorithm for our noise reduction technique involves putting a virtual counter on each surface lattice site that registers a positive count m every time a new atom from the incident beam randomly drops on that counter. A real deposition event is allowed in the simulation only when the counter registers a predetermined value of m (for example, $m=5$ or 7, etc.) No actual deposition or incorporation occurs at that particular site until m reaches the predetermined noise reduction factor number (the same is, of course, true for each surface site). For $m=1$, a noise reduction factor of unity, the growth model by definition is the same as the usual growth simulation without any noise reduction. Note that a large value of the noise reduction parameter m (10–20, typically) does not help the situation because it suppresses kinetic surface roughening far too well producing smooth layer-by-layer growth with essentially no roughening. The optimal value of m (usually in the range 3–10) has to be chosen by trial and error for specific growth models—m cannot be too small for noise effects will then dominate with steep hills and deep valleys (large values of surface height gradients), and m cannot be too large since the smooth layer-by-layer growth will then proceed indefinitely with no kinetic surface roughening. We use $m=1$–20 for the results shown in this paper.

We demonstrate in Fig. 2 the main qualitative features of noise reduction technique by showing our simulated $d=1+1$ dimensional growth morphologies (i.e., for growth on one-dimensional substrate). In Fig. 2(a) we show our simulated growth morphologies without any noise reduction (i.e., $m=1$) for the four models (see Fig. 1) studied in this paper (i.e., DT, WV, ADT, and SWV) whereas the corresponding noise-reduced growth morphologies (with the noise reduction factor $m=5$) are shown in Fig. 2(b)—note the very different vertical height scales in the two sets of figures, demonstrating clearly the qualitative feature of the noise reduction induced suppression of high hills and deep valleys in Fig. 2(b) compared with Fig. 2(a). We point out (as discussed in details in Ref. [2]) that the universality class of a growth model is known to be unaffected by the noise reduction technique, which only serves to suppress corrections to scaling.

Before presenting our simulation results, we briefly describe the continuum dynamical growth equation approach that we use in discussing the universality class of various models. Denoting the dynamical height fluctuation variable as h, and the lateral coordinate (along the substrate) as $\mathbf{x}$ and the growth "time" as t (note that the growth time is defined entirely by the average deposition rate since we neglect bulk

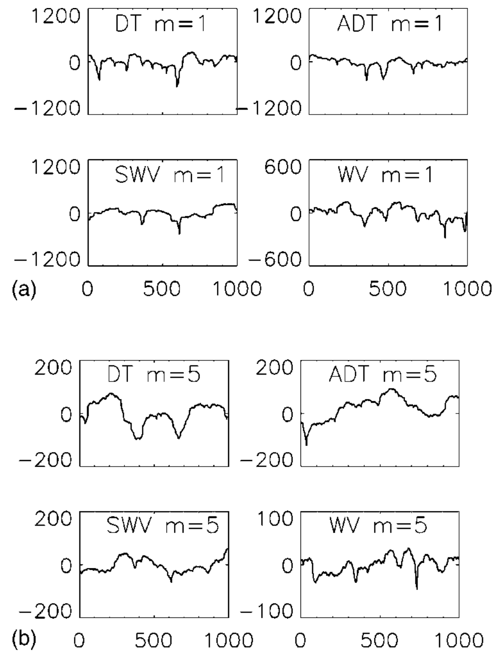

FIG. 2. (a) Typical growth morphologies for one dimensional ($d=1+1$) growth without noise reduction in the four models (DT, ADT, SWV, WV) as shown in the paper for a substrate of a 1000 site section from a 10 000 site substrate after the deposition of 10^6 monolayers (ML) (the average height is subtracted out for clarity); (b) the same as in (a) but with a noise reduction factor $m=5$ [note the very different vertical scales in (b) compared with (a)].

vacancies, overhangs, and evaporation in our SOS model), the most general leading-order growth equation [1] for our problem can be written as

$$\frac{\partial h}{\partial t}=\nu_2\nabla^2 h-\nu_4\nabla^4 h+\lambda_{22}\nabla^2(\boldsymbol{\nabla} h)^2+\eta, \qquad (1)$$

where $h\equiv h(\mathbf{x},t)$, and η is the spatiotemporal Gaussian white noise associated with the incident beam shot noise. The symmetry-allowed fourth-order nonlinear term $\boldsymbol{\nabla}(\boldsymbol{\nabla} h)^3$ has been left out of Eq. (1) since, upon renormalization, it generates the linear $\nabla^2 h$ term already included in Eq. (1). [The constant flux term associated with the average growth has been left out of Eq. (1) since the height fluctuation variable h is defined with respect to the average interface position.] When $\nu_2\neq 0$, the asymptotic growth universality class is the so-called Edwards-Wilkinson (EW) universality ($\nu_2>0$) or the "unstable" or mounded growth universality ($\nu_2<0$)—the fourth-order terms in Eq. (1) are then all irrelevant although they may be important in controlling the nonasymptotic transient regime that may last for a long time. When $\nu_2\equiv 0$ (as well as the corresponding fourth-order nonlinear term $\boldsymbol{\nabla}(\boldsymbol{\nabla} h)^3$ being absent), the asymptotic growth universality class is the MBE growth universality determined by the $\nabla^2(\boldsymbol{\nabla} h)^2$ term with the initial transient regime (which could be extremely long lived depending on the ratio ν_4/λ_{22}) controlled by the irrelevant fourth-order linear term. When both $\nu_2=0$ and $\lambda_{22}=0$, the growth universality class is the so-called Mullins-Herring (MH) universality defined

TABLE I. Summary of our results for the DT and WV models.

Model	Properties	$d=1+1$	$d=2+1$
	Exponents	$\alpha=1,\beta=1/3,z=3$	$\alpha=0(\log),\beta=0(\log),z=2$
DT	Morphology	Kinetically rough	Flat and very smooth
	Particle current	$J=0(\sim\pm10^{-6})$	$J\sim-10^{-4}$
	Exponents	$\alpha=1/2,\beta=1/4,z=2$	$\alpha=1,\beta=1/3,z=4$
WV	Morphology	Kinetically rough	Mounds with selected slope
	Particle current	$J\sim-10^{-3}$	$J_{100}\sim-10^{-2}$; $J_{111}\sim+10^{-2}$

by the $\nu_4\nabla^4 h$ term. The various growth exponents for these universality classes are well known and can be found in the literature [1] where the details on EW, MH, and MBE universalities are also discussed.

Determining the asymptotic growth universality class of a discrete nonequilibrium growth model may be difficult due to extremely long-lived transients that would mask the crossover behavior. This is obviously a more acute problem when any two (or all three) of the growth terms $(\nu_2,\lambda_{22},\nu_4\neq 0)$ are nonzero in Eq. (1), although complications could also arise from higher-order terms (i.e., higher than fourth order) neglected in Eq. (1). It is reasonable to assume that in the generic situation, i.e., in the absence of any compelling symmetry or conservation law induced constraints, the fourth-order nonlinear equation, Eq. (1), should suffice to determine the asymptotic universality class of a given growth model, and there is no need to consider a higher-order (e.g., the sixth-order) dynamical equation. This is particularly true since simple power counting considerations indicate that many of the higher-order nonlinearities produce the ν_2 or the λ_{22} term upon renormalization, and most of the higher-order terms are simply irrelevant. Thus, it is quite possible that even in the extremely unlikely situation that all the terms in Eq. (1) vanish, i.e., $\nu_2,\lambda_{22},\nu_4=0$, for some pathological reasons Eq. (1) may still remain a valid generic description of the asymptotic growth universality class, because higher-order nonlinearities neglected in Eq. (1) give rise to the terms $\nabla^2 h$ and/or $\nabla^2(\boldsymbol{\nabla} h)^2$ in the growth dynamics.

We have determined the asymptotic universality class from our discrete stochastic simulations by calculating the growth exponents characterizing the height-height correlation functions as well as by measuring the surface current on a tilted substrate as proposed in Ref. [6]. We emphasize that the noise reduction technique is crucial in enabling us to conclusively determine the asymptotic universality class of the four (DT, WV, ADT, SWV) discrete growth models we study in contrast to earlier simulational studies that have been severely hampered by slow crossover, long transient, and strong correction to scaling problems. Our conclusions about the growth universality classes of the four models, as summarized in Table I, are based on consistent results from simultaneous measurements of growth exponents and surface currents on tilted substrates. All our exponent calculations are results of extensive averaging over many simulation runs (and each run is self-averaging since we average over all the substrate lattice sites)—typically we use 100 runs for determining our exponents although 10 runs are usually adequate.

The results in $d=1+1$ for the DT and the WV model are already known in the literature [2,3,5,6] from earlier extensive one-dimensional simulations of these two well-studied models. The one-dimensional DT (WV) model belongs to MBE (EW) asymptotic universality as we also confirm decisively in our current simulations. The fact that the $d=1+1$ DT model has $\nu_2=0$ exactly follows from a hidden dynamical symmetry [7] in the DT growth rules, which are *symmetric*, i.e., the DT diffusion rules do not have any preference between two- and three-bonded incorporation sites. This also makes the current on a tilted substrate to vanish exactly in the $d=1+1$ DT model, confirming that it is generically in the MBE universality class [$\nu_2=0$ in Eq. (1)]. Similarly, earlier tilted substrate current measurements [6] as well as direct simulational exponent measurements for the $d=1+1$ WV model establish it to be in the EW universality class (i.e., $\nu_2>0$) although the WV model shows very similar scaling behavior to the DT model for a very long transient regime since $|\nu_2|\approx 0$ and ν_4,λ_{22} are rather large in the WV model. Our noise reduced simulations of the WV model [2] verify rather strikingly that this model belongs to EW universality in $1+1$ dimensions. The suppression of crossover and correction to the scaling effect in the $(1+1)$-dimensional DT and WV models was our original motivation for introducing [2] the noise reduction technique in this context.

The kinetically rough surface growth is traditionally analyzed in terms of the dynamic scaling hypothesis where the surface width (the root mean square fluctuation in the surface height) or more generally, the height-height correlation function shows generically scale invariant power-law scaling behavior, with critical exponents $\alpha,\beta,z=\alpha/\beta$, given by $W(L,t)\sim L^{\alpha}f(\xi(t)/L)$ where W is the dynamical surface width at growth time t for a substrate of lateral size L and $\xi(t)$ is the lateral dynamical correlation length for the specific growth process with $\xi(t)\sim t^{1/z}$. The scaling function $f(x)$ behaves as $f(x\gg 1)\sim 1$ and $f(x\ll 1)\sim x^{\alpha}$ so that $W(L,t\to\infty)\equiv W_s(L)\sim L^{\alpha}$ and $W(L,t\ll L^{1/z})\sim t^{\beta}$ with $\beta=\alpha/z$, where $W_s(L)$ is the saturated steady-state long time surface width and $W(L\ll\xi(t))$ is the pre-steady-state dynamical surface width. The set of critical exponents α, β, and $z=\alpha/\beta$ define the growth universality class, and, in principle, can be determined for a particular continuum growth equation. For the sake of completeness (see the WV growth results presented below), we point out that for mounding instability with slope selection, when the surface morphology evolves into a regular mounded pattern with the sides of the mounds having constant slopes, one gets $\alpha\equiv 1$,

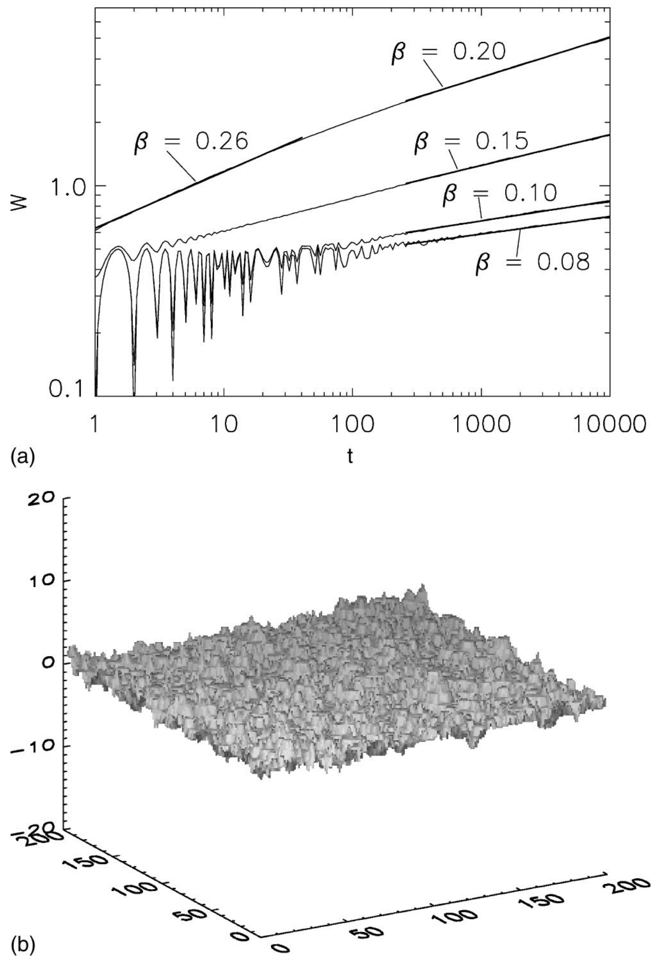

FIG. 3. (a) Interface width W versus time t in the $2+1$ DT model with substrate size $L=1000\times1000$ and noise reduction factor $m=1$, 3, 10, and 15 from top to bottom. The solid lines are best power-law fit, which yield the growth exponent β that decreases as m increases. (b) A typical morphology of a noise reduced DT model $(m=5)$ from a 1000×1000 substrate (only a section of 200×200 is shown here) at 400 ML. The interface width is measured in units of monolayers or lattice spacing and time is measured in units of number of deposited monolayers (i.e., average height of the surface).

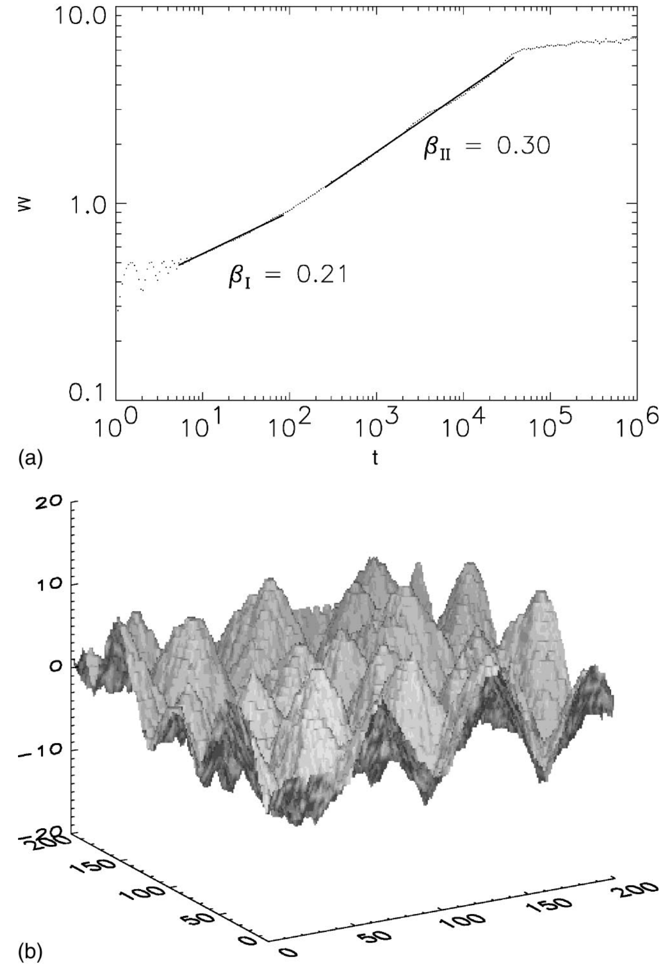

FIG. 4. (a) Interface width W versus time t in the $2+1$ WV model with substrate size $L=100\times100$ and noise reduction factor $m=5$. (b) A typical morphology of a noise reduced WV model $(m=5)$ from a 500×500 substrate (only a section of 200×200 is shown here) at 10^6 ML. The units are dimensionless as explained in the caption for Fig. 3.

and only one exponent (β or z) $\beta=1/z$ then defines the growth pattern. In Table I and in Figs. 3 and 4 we present our calculated critical exponents for the DT and WV model along with the associated continuum equation descriptions.

Our most surprising findings are presented in Fig. 3 (for the DT model) and Fig. 4 (WV model) where our results for the $(2+1)$-dimensional DT and WV models are depicted. We find that the DT (WV) model in $(2+1)$ dimensions belongs to the EW (unstable) dynamic universality in contrast to the $(1+1)$-dimensional universality class of these models. This is an important result presented in this paper that disagrees with the earlier conclusions in the literature. The determination of the asymptotic universality class of the $(2+1)$-dimensional DT and WV model is the main result being presented in this paper.

We first discuss the $(2+1)$-dimensional DT results, which are in some sense less surprising than the corresponding WV results because EW universality is the generic SOS universality class. Our measurement of current on tilted substrates in the $(2+1)$-dimensional noise reduced DT simulations always exhibit a small but finite downhill current indicating the presence of a small $\nu_2>0$ term in Eq. (1). This should be contrasted with the corresponding $(1+1)$-dimensional DT results where a simple symmetry argument as well as extensive numerical simulations definitively establish the absence of the $\nabla^2 h$ term in the $d=1+1$ DT growth equation, i.e., $\nu_2=0$ $(\neq 0)$ in the $d=1+1(2+1)$ DT model. The asymptotic universality class of $(1+1)$-dimensional DT model is now well established to be given by the following continuum growth equation, which does *not* have the generic $\nu_2\partial^2 h/\partial x^2$ term of Eq. (1):

$$\frac{\partial h}{\partial t}=\nu_4\frac{\partial^4 h}{\partial x^4}+\lambda_{22}\frac{\partial^2}{\partial x^2}\left(\frac{\partial h}{\partial x}\right)^2+\sum_{n=4,6,\ldots}\lambda_{2n}\frac{\partial^2}{\partial x^2}\left(\frac{\partial h}{\partial x}\right)^n+\eta, \tag{2}$$

where the "higher-order" terms of the form $\partial^2/\partial x^2(\partial h/\partial x)^{2m}$ with $2m \equiv n = 4,6,8,\ldots$, are marginally relevant in $d=1+1$ (i.e., simple power counting reveals them as having the same anomalous dimensions as the nonlinear fourth-order λ_{22} term). Our tilted substrate current measurement in $d=2+1$ DT model shows the existence of a small slope-dependent downhill current of appoximate magnitude $\sim 10^{-2}$, whereas the corresponding $d=1+1$ DT model has a current $\sim 10^{-6}$ (of random sign), which is indistinguishable from the background noise effect. We, therefore, conclude that the $(2+1)$-dimensional DT model, describing nonequilibrium growth on physical surfaces and interfaces, belongs to the generic EW universality class, and has the following coarse-grained continuum description:

$$\frac{\partial h}{\partial t} = \nu_2 \nabla^2 h - \nu_4 \nabla^4 h + \sum_{n=1,2,3,\ldots} \lambda_{2(2n)} \nabla^2 (\boldsymbol{\nabla} h)^{2n} + \eta. \tag{3}$$

Our finding that ν_2 is very small, but nonzero, in Eq. (3) for $(2+1)$-dimensional DT growth should not come as a big surprise (except, of course, for the fact that it has not earlier been discovered in the literature including our own earlier work on the DT model) [8] because the vanishing of ν_2 in the $(1+1)$-dimensional DT model arises from a rather peculiar kinetic-topological symmetry of the DT model, which applies only in one dimension and cannot be generalized to two-dimensional surfaces. In the absence of a compelling symmetry argument manifestly making $\nu_2=0$ in the growth equation, one should expect its generic presence in the $(2+1)$-dimensional DT model although the extreme quantitative smallness of ν_2 has made it difficult so far to establish its finiteness in simulations. Our finding that there is a small downhill current on tilted substrates in the $(2+1)$-dimensional DT model and that the critical exponents of $(2+1)$-dimensional DT growth (Fig. 3) are consistent with EW universality class [and *not* particularly consistent with the MBE universality defined by $\nu_2=0$ in Eq. (1)] leads to the conclusion that $(2+1)$-dimensional DT growth is in the EW universality class [$\nu_2 \neq 0$ in Eq. (1)] and $(1+1)$-dimensional DT growth is in the MBE universality class ($\nu_2=0$). We have also carried out DT simulations in (unphysical) $(3+1)$ dimensions finding very good agreement with EW universality properties.

Our results for the $(2+1)$-dimensional WV model (Fig. 4) are very dramatic and completely unanticipated. We find that the $(2+1)$-dimensional WV model leads to a spectacular quasiregular mounded morphology indicating unstable epitaxial growth (whereas the corresponding $(1+1)$-dimensional WV growth is asymptotically stable and flat, belonging to the EW universality class). Thus the usual critical exponents ($\alpha,\beta,z=\alpha/\beta$) are not particularly meaningful for $(2+1)$-dimensional WV growth (although they can still be defined in the simulation results—the exponents, however, provide a misleading picture since the growth front, instead of being statistically scale invariant as it should be for kinetic surface roughening exhibiting power laws controlled by critical exponents, has a quasiregular mounded pattern).

The tilted substrate current measurement in the $(2+1)$-dimensional WV growth yields another curious surprise. It turns out that the local current is uphill [along (111) plane] or downhill [along (100)] depending on how (i.e., along which direction) one decides to tilt. The fact that the tilted substrate current could depend sensitively (i.e., stabilizing in some directions and destabilizing in other directions) on the tilt direction has not earlier been reported in the literature where most reported current measurements are carried out in $1+1$ dimensions (where, of course, this problem cannot arise) with the hope or expectation (proven to be false in this paper) that an accurate determination of the universality class of a growth model in $1+1$ dimensions will automatically give us the *same* universality class in $2+1$ dimensions—WV model is in the EW universality class in $1+1$ dimensions and unstable (mounded morphology) in $2+1$ dimensions. Thus the tilted substrate current measurement, while being capable of providing the correct universality class in $1+1$ dimensions, may very well lead to misleading and wrong conclusions in higher dimensions where the current on tilted substrates is explicitly direction dependent and is not uniquely defined. We will publish quantitative details on the WV mounding phenomenon elsewhere. Here we point out the following observations.

(1) The underlying mechanism for the WV mounded morphology is related to surface cluster-edge (or kink) diffusion induced mounding recently discussed in the literature [9].

(2) This WV mounding phenomenon, arising as it does from kinetic-topological aspect of surface diffusion, leads to very strong instabilities in (unphysical) dimensions higher than $2+1$, where early work reported [10] unexplained strong mound formation in $(3+1)$- and $(4+1)$-dimensional WV growth. We have carried out WV simulations in $3+1$ dimensions, finding very strong mounding even without any noise reduction, consistent with earlier findings [10].

Finally we consider the two intermediate models, ADT and SWV, for the sake of completeness. The $d=2+1$ growth morphologies in these two models are shown in Figs. 5 (ADT) and 6 (SWV). Without presenting the actual numerical results for the critical exponents for these two models, we just mention that the ADT and SWV models in $2+1$ dimensions behave qualitatively similar to the WV model in $2+1$ dimensions with fairly strong mounding under noise reduction although the morphological details in the two models differ somewhat with the SWV morphology being similar to the pyramidal structures of the WV morphology and the ADT morphology having flat top mounds with very deep and narrow grooves. Thus, ADT/SWV/WV all have unstable growth in $2+1$ (or higher) dimensions with quasiregular, mounded morphology, whereas DT in $2+1$ (or higher) dimensions is in the EW universality with stable and smooth growth morphology. All four models have kinetically rough statistically scale invariant growth in $1+1$ dimensions with the WV (DT) model belonging to EW (MBE) universality class.

Before concluding we point out that the noise reduction technique is only one of several methods one can apply to improve the calculation of exponents in kinetically rough surface growth. For example, single-step model and re-

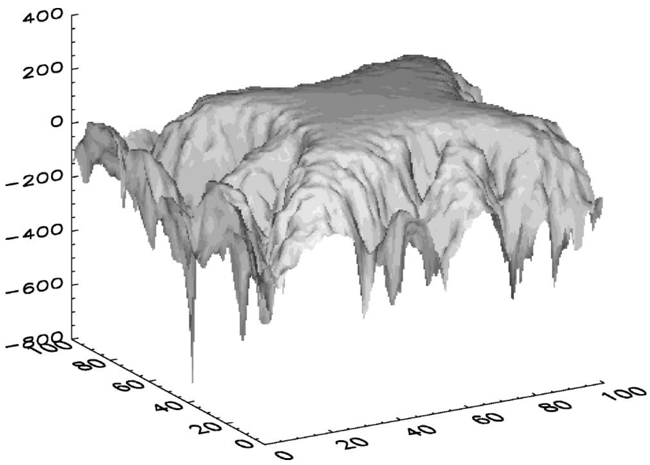

FIG. 5. A typical morphology of a noise-reduced ADT model ($m=5$) from a 100×100 substrate at 10^6 ML.

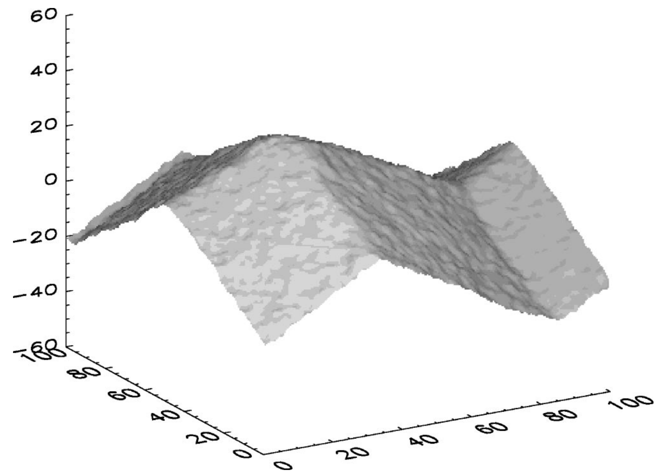

FIG. 6. A typical morphology of a noise reduced SWV model ($m=5$) from a 100×100 substrate at 10^6 ML.

stricted solid-on-solid models [1] are successful in reducing noise in ballistic deposition simulations. These techniques are, in fact, alternative ways of suppressing large surface height differences, very similar in spirit to our noise reduction technique.

We conclude by stating that we have found the universality class concept to be of limited usefulness in conserved discrete limited mobility nonequilibrium surface growth models. The same growth rules defining a particular model (e.g., WV or DT) may belong to different universality classes in different dimensionalities (not in the sense of superuniversality, but in a more fundamental nontrivial sense as if an equilibrium model, which is in the Ising universality class in two dimensions, behaves as an *x*-*y* model in three dimensions—a patently absurd notion). In addition, rather minor changes in local growth rules could lead to dramatic differences in the resulting growth morphology or the universality class—DT and WV have very similar local growth rules, but their morphologies, smooth (DT) and mounded (WV), and universality class (EW for DT and unstable for WV) are strikingly qualitatively different. We also find that measuring surface current on tilted substrates [6], while being a potentially useful technique for discussing the $(1+1)$-dimensional universality class, may not work in $2+1$ dimensions and may produce misleading or conflicting conclusions depending on the precise direction of the surface current. We have assumed throughout the paper that the noise reduction technique, which is absolutely crucial in our obtaining the asymptotic universality classes of various models we study, does not modify the universality class of a growth model (and only suppresses transient and correction-to-scaling effects by reducing the effective noise strength). This belief is based on extensive earlier analysis of the noise reduction technique in the literature, which, in general, is thought not to affect the growth universality class.

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